

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2};$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2};$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2};$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2}.$$

Задача 1 Упростить выражение

$$\left(\sin \left(\alpha + \frac{\pi}{12} \right) + \sin \left(\alpha - \frac{\pi}{12} \right) \right) \sin \frac{\pi}{12}.$$

С помощью первой формулы получаем:

$$\begin{aligned} & \left(\sin \left(\alpha + \frac{\pi}{12} \right) + \sin \left(\alpha - \frac{\pi}{12} \right) \right) \sin \frac{\pi}{12} = \\ & = 2 \sin \alpha \cos \frac{\pi}{12} \sin \frac{\pi}{12} = \frac{1}{2} \sin \alpha. \end{aligned}$$

14. Верно ли равенство:

а) $\sin \frac{7\pi}{12} - \sin \frac{\pi}{12} = \frac{\sqrt{2}}{2};$

б) $\cos \frac{11\pi}{24} - \cos \frac{\pi}{8} = -\sin \frac{7\pi}{24};$

в) $\sin \frac{11\pi}{18} + \sin \frac{7\pi}{18} = \cos \frac{2\pi}{9};$

г) $\cos \frac{5\pi}{8} + \cos \frac{\pi}{8} = \sqrt{2} \cos \frac{3\pi}{8} ?$

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|-----------------------------|----|----------------------|----------------------|----------------------|-----------------|-----------------------|-----------------------|-----------------------|-------|-----------------------|-----------------------|-----------------------|------------------|-----------------------|-----------------------|-----------------------|--------|
| α | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° | 210° | 225° | 240° | 270° | 300° | 315° | 330° | 360° |
| $\sin \alpha$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | -1 | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{1}{2}$ | 0 |
| $\cos \alpha$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | -1 | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\operatorname{tg} \alpha$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | - | $-\sqrt{3}$ | -1 | $-\frac{\sqrt{3}}{3}$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | - | $-\sqrt{3}$ | -1 | $-\frac{\sqrt{3}}{3}$ | 0 |
| $\operatorname{ctg} \alpha$ | - | $\sqrt{3}$ | 1 | $\frac{\sqrt{3}}{3}$ | 0 | $-\frac{\sqrt{3}}{3}$ | -1 | $-\sqrt{3}$ | - | $\sqrt{3}$ | 1 | $\frac{\sqrt{3}}{3}$ | 0 | $-\frac{\sqrt{3}}{3}$ | -1 | $-\sqrt{3}$ | - |
| α | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | 2π |